# Heat transfer: conduction (in steady and unsteady state), extending surfaces, convection + radiation

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# CONVECTIVE HEAT TRANSFER

Convection involves the transfer of heat by the motion and mixing of ‘macroscopic" portions of a fluid (that is, the flow of a fluid past a solid boundary). The term natural convection is used if this motion and mixing is caused by density variations resulting from temperature differences within the fluid. The term forced convection is used if this motion and mixing is caused by an outside force, such as a pump. The transfer of heat from a hot water radiator to a room is an example of heat transfer by natural convection. The transfer of heat from the surface of a heat exchanger to the bulk of a fluid being pumped through the heat exchanger is an example of forced convection.

Heat transfer by convection is more difficult to analyze than heat transfer by conduction because no single property of the heat transfer medium, such as thermal conductivity, can be defined to describe the mechanism. Heat transfer by convection varies from situation to situation (upon the fluid flow conditions), and it is frequently coupled with the mode of fluid flow. In practice, analysis of heat transfer by convection is treated empirically (by direct observation).

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Convection heat transfer is treated empirically because of the factors that affect the stagnant film thickness:

- Fluid velocity
- Fluid viscosity
- Heat flux
- Surface roughness
- Type of flow (single-phase/two-phase)

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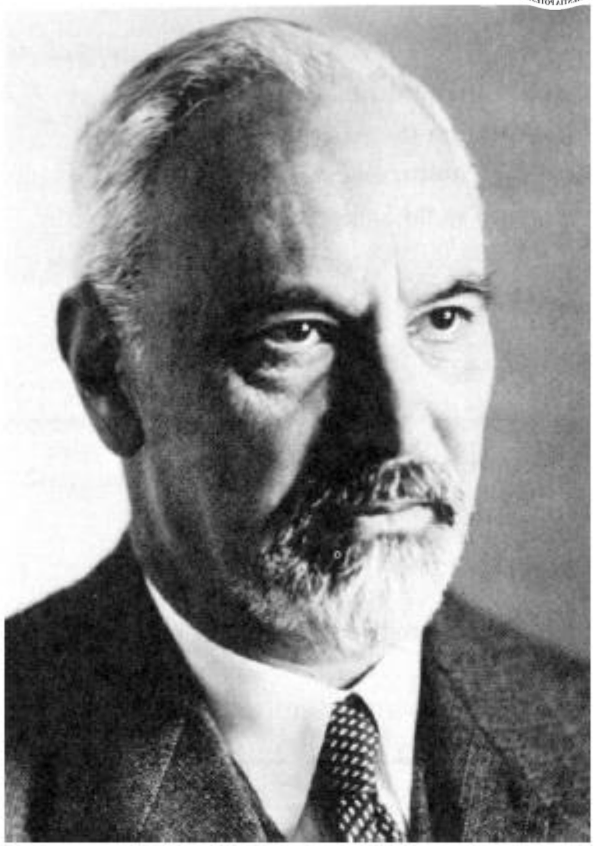
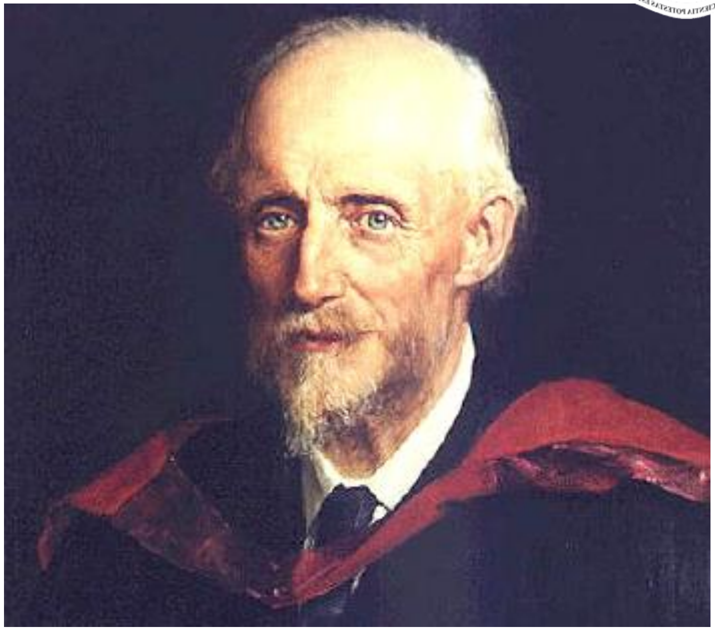
$$\mathrm{Pr} = 3600 \cdot \frac{\nu}{\alpha}$$



Ludwig Prandtl (1875–1953).

$$Re = \frac{w \cdot d \cdot \gamma}{\eta \cdot g}$$



Osborne Reynolds (1842 to 1912)
Reynolds was born in Ireland but he taught at the University of Manchester. He was a significant contributor to the subject of fluid mechanics in the late 19th C.

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$$Nu = \frac{\alpha \cdot d}{\lambda}$$

Ernst Kraft Wilhelm Nusselt (1882–1957). This photograph, provided by his student, G. Lück, shows Nusselt at the Kesselberg waterfall in 1912. He was an avid mountain climber.

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# CONDUCTIVE HEAT TRANSFER

Conduction involves the transfer of heat by the interaction between adjacent molecules of a material. Heat transfer by conduction is dependent upon the driving "force" of temperature difference and the resistance to heat transfer. The resistance to heat transfer is dependent upon the nature and dimensions of the heat transfer medium. All heat transfer problems involve the temperature difference, the geometry, and the physical properties of the object being studied.

In conduction heat transfer problems, the object being studied is usually a solid. Convection problems involve a fluid medium. Radiation heat transfer problems involve either solid or fluid surfaces, separated by a gas, vapor, or vacuum.

There are several ways to correlate the geometry, physical properties, and temperature difference of an object with the rate of heat transfer through the object. In conduction heat transfer, the most common means of correlation is through Fourier's Law of Conduction. The law, in its equation form, is used most often in its rectangular or cylindrical form

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$$q_s = - \lambda \, dt / dx, \; (W / m^2)$$

*The effects of heat are subject to constant laws which cannot be discovered without the aid of mathematical analysis. The object of the theory which we are about to explain is to demonstrate these laws; it reduces all physical researches on the propagation of heat to problems of the calculus whose elements are given by experiment.*

***The Analytical Theory of Heat, J. Fourier, 1822***

Baron Jean Baptiste Joseph Fourier (1768–1830).

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$$\frac{1}{a}\frac{\partial t}{\partial \tau} = \nabla^2 t + \frac{q_v}{\lambda}$$

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

$$\nabla^2 t = \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2} + \frac{\partial^2 t}{\partial z^2}$$

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# RADIATIVE HEAT TRANSFER

Radiant heat transfer involves the transfer of heat by electromagnetic radiation that arises due to the temperature of a body. Most energy of this type is in the infra-red region of the electromagnetic spectrum although some of it is in the visible region. The term thermal radiation is frequently used to distinguish this form of electromagnetic radiation from other forms, such as radio waves, x-rays, or gamma rays. The transfer of heat from a fireplace across a room in the line of sight is an example of radiant heat transfer.

Radiant heat transfer does not need a medium, such as air or metal, to take place. Any material that has a temperature above absolute zero gives off some radiant energy. When a cloud covers the sun, both its heat and light diminish. This is one of the most familiar examples of heat transfer by thermal radiation.

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The **Stefan–Boltzmann law** is an example of a power law.

The law was deduced by **Jožef Stefan** in 1879 on the basis of experimental measurements made by John Tyndall and was derived from theoretical considerations, using thermodynamics, by Stefan's student **Ludwig Boltzmann** in 1884. Boltzmann treated a certain ideal heat engine with the light as a working matter instead of the gas. The law is valid only for ideal black objects, the perfect radiators, called black bodies. Stefan published this law on March 20 in the article *Über die Beziehung zwischen der Wärmestrahlung und der Temperatur* (*On the relationship between thermal radiation and temperature*) in the *Bulletins from the sessions* of the Vienna Academy of Sciences.

**Jožef Stefan**
**(1835-1893)**

$$Q = C\left(\frac{T}{100}\right)^4, \quad (W/m^2)$$

**Ludwig Boltzmann**
**(1844-1906)**

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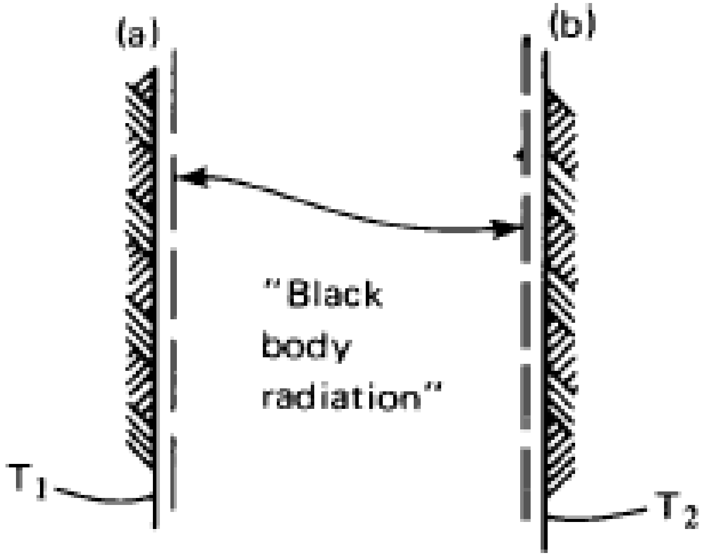
## Two parallel gray bodies




$$C_{1,2} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} - \frac{1}{5{,}77}}$$

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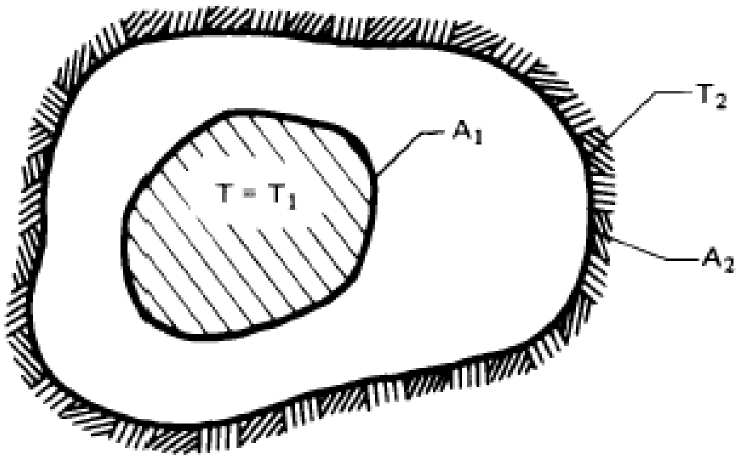
# One gray body enclosed by another




$$C_{1,2} = \frac{1}{\frac{1}{C_1} + \frac{A_1}{A_2}\left(\frac{1}{C_2} - \frac{1}{5,77}\right)}$$

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# APLICATIONS
# OF
# CONDUCTIVE HEAT TRANSFER
# IN
# HEATED ENCLOSURES

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-Heat transfer in heated walls

-Heat transfer inside the parts

-Extending surfaces

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# Heat transfer in heated walls

**Poisson equation, in steady regime:**

$$\nabla^2 t + \frac{q_v}{\lambda} = 0$$

**Laplace equation, in steady regime and without internal heating sources**

$$\nabla^2 t = 0$$

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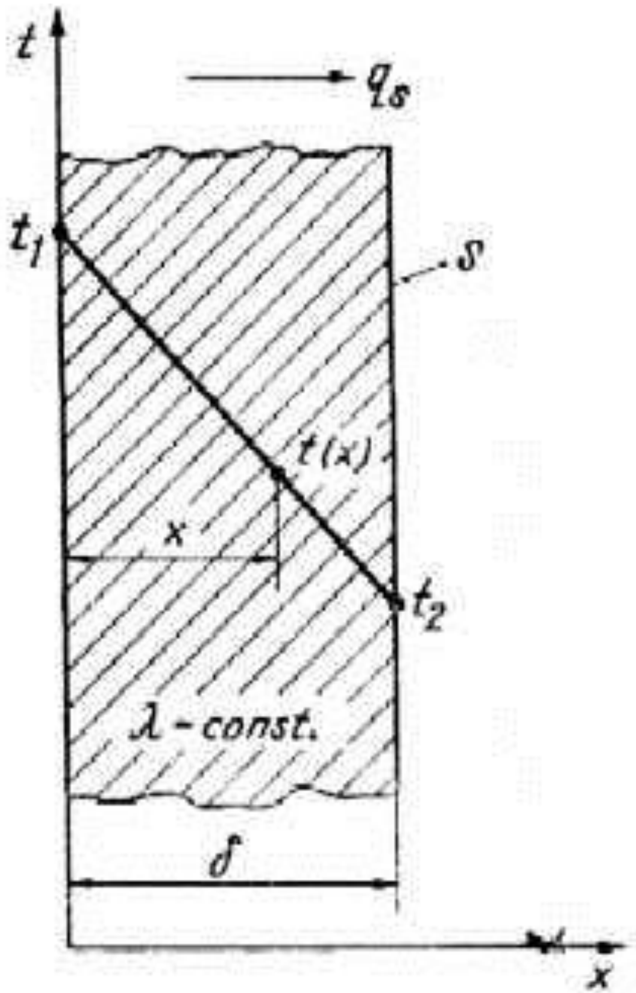
**For heat transfer in walls it consider Laplace equation, in steady regime and without internal heating sources, in one-dimensional space.**




$$\nabla^2 t = \frac{d^2 t}{dx^2}$$

$$\frac{d^2 t}{dx^2} = 0 \Rightarrow \frac{dt}{dx} = C_1 \Rightarrow dt = C_1 dx$$

at $x = 0$, $t = t_1$,

at $x = \delta$, $t = t_2$.

It get:

$C_1 = -(t_1 - t_2)/\delta$; $C_2 = t_1$,

Finally:

$t(x) = t_1 - (t_1 - t_2)\, x/\delta$.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$$\nabla^2 t = \frac{d^2 t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = 0$$

$$r\frac{d^2 t}{dr^2} + \frac{dt}{dr} = r\frac{d}{dr}\left(\frac{dt}{dr}\right) + 1\frac{dt}{dr} = \frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0 \Rightarrow r\frac{dt}{dr} = C_1 \Rightarrow dt = \frac{C_1}{r}dr \Rightarrow t(r) = C_1 \ln r + C_2$$

- At $r = r_1$, $t = t_1$;
- At $r = r_2$, $t = t_2$,

$$C_1 = \frac{t_1 - t_2}{\ln\frac{r_1}{r_2}}; C_2 = t_1 - (t_1 - t_2)\frac{\ln r_1}{\ln\frac{r_1}{r_2}}$$

$$t(r) = t_1 - (t_1 - t_2)\frac{\ln\frac{r}{r_1}}{\ln\frac{r_2}{r_1}} = t_1 - (t_1 - t_2)\frac{\ln\frac{d}{d_1}}{\ln\frac{d_2}{d_1}}$$

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Finally, these are the equations for rectangular walls and cylindrical ones:

$$q_s = \frac{\lambda(t_1 - t_2)}{\delta}$$

$$q_1 = \frac{2\pi\lambda(t_1 - t_2)}{\ln\frac{d_2}{d_1}}$$

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## Equivalent Resistance Method

It is possible to compare heat transfer to current flow in electrical circuits. The heat transfer rate may be considered as a current flow and the combination of thermal conductivity, thickness of material, and area as a resistance to this flow. The temperature difference is the potential or driving function for the heat flow, resulting in the Fourier equation being written in a form similar to Ohm's Law of Electrical Circuit Theory. If the thermal resistance term **Δx/λ** is written as a resistance term where the resistance is the reciprocal of the thermal conductivity divided by the thickness of the material, the result is the conduction equation being analogous to electrical systems or networks. The electrical analogy may be used to solve complex problems involving both series and parallel thermal resistances.

A typical conduction problem in its analogous electrical form is given in the following example, where the "electrical" Fourier equation may be written as follows.

$$I = \Delta U / R_e; \qquad q_s = \Delta t / R.$$

$$Q_{p \cdot p \cdot c} = \frac{\Delta t}{\sum_{i=1}^{n} \frac{d_i}{\lambda_i}}$$

$$Q_{p \cdot p \cdot c} = 2 \cdot \pi \cdot l \cdot \frac{t_1 - t_{n+1}}{\sum \frac{1}{\lambda_i} \cdot \ln \frac{r_{i+1}}{r_i}}$$

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# Heat transfer in heated parts

**General equation, in unsteady regime:**

$$\frac{1}{a}\frac{\partial t}{\partial \tau} = \nabla^2 t + \frac{q_v}{\lambda}$$

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In this case is very hard to solve an equation like this. So, it consider the balance heat equation :

$$Q = \alpha S(t - t_\infty) = -c_p \rho V \frac{dt}{d\tau}, \qquad \frac{dt}{d\tau} = -\frac{\alpha S(t - t_\infty)}{\rho c_p V},$$

The initial conditions are : t = $t_0$ at $\tau = 0$.

The solution is : $\frac{\theta}{\theta_0} = \frac{t - t_\infty}{t_0 - t_\infty} = e^{-\frac{\alpha S \tau}{\rho c_p V}}$

$$\frac{\theta}{\theta_0} = \exp\left(-\frac{\alpha x_1^2 \tau}{\rho c_p x_1^3}\right) = \exp\left(-\frac{\alpha x_1}{\lambda} \frac{\lambda \tau}{\rho c_p x_1^2}\right) = \exp(-Bi \cdot Fo)$$

This relation can permit finding $\alpha$ or time $\tau$.

Further, it can define the criteria's :

**Fourier:** $Fo = a\tau/x_1^2$ **Biot:** $Bi = \alpha x_1/\lambda$

For larger parts, the Fo criteria gives no reasonable interpretation. So. it is used another criteria, **Boussineq**: $Bq = Fo \cdot (Bi)^2 = \frac{a \cdot \tau}{R^2} \cdot \frac{\alpha^2 \cdot R^2}{\lambda^2} = \frac{\alpha^2}{\lambda^2} \cdot a \cdot \tau$

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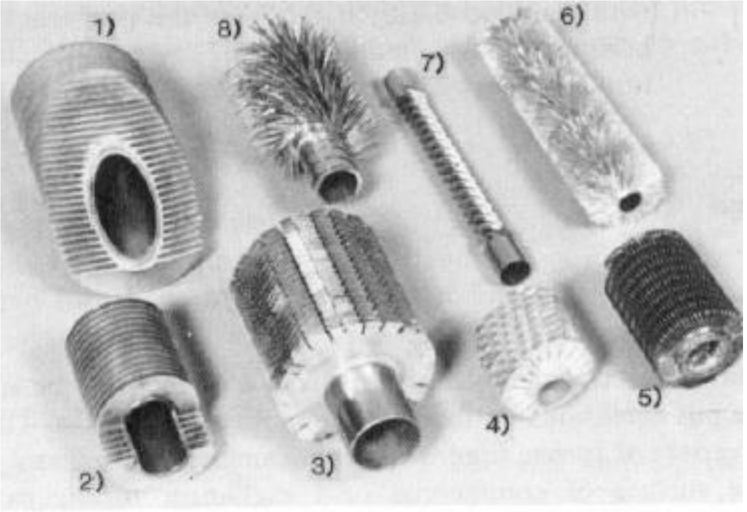
# Extending surfaces: fin design

**The purpose of fins**

The convective removal of heat from a surface can be substantially improved if we put extensions on that surface to increase its area. These extensions can take a variety of forms. Figures, for example, shows many different ways in which the surface of commercial heat exchanger tubing can be extended with protrusions of a kind we call *fins.*



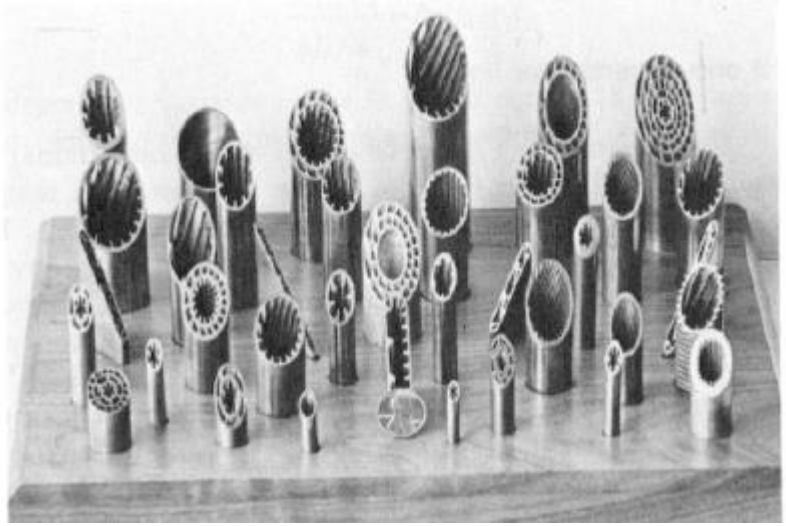



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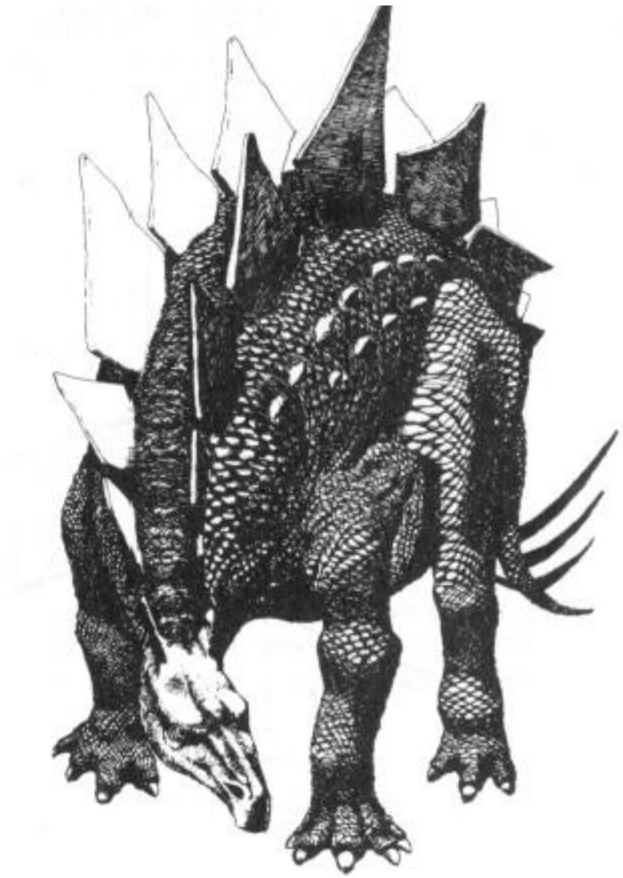
Figure shows another very interesting application of fins in a heat exchanger design. This picture is taken from an issue of *Science Magazine*, which presents an intriguing argument by Farlow, Thompson, and Rosner. They offered evidence suggesting that the strange rows of fins on the back of the *Stegosaurus were used to shed excess body heat* after strenuous activity, which is consistent with recent suspicions that *Stegosaurus was warm-blooded.*
These examples involve some rather complicated fins. But the analysis of a straight fin protruding from a wall displays the essential features of all fin behavior. This analysis has direct application to a host of problems.



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The extension of heat exchange surface is used as an intensification method of convection processes for the fluids that realize small convection coefficients like gases. According to Newton relation $Q = \alpha S\ (t_p - t_f)$, for a given difference between the wall and fluid temperature and a low value of $\alpha$ coefficient, the increase of heat flow Q can be done by increasing surface S.

The extension of heat exchange surface makes with the help of some fins with different geometric shapes (longitudinal, radial, acicular, etc.) attached to a support surface (basic) made of the same material with support wall or of different materials.

The calculus is based on the following simplifier hypotheses:

1. **thermal regime is constant in time;**
2. **thermal conductivity of fins material $\lambda$ = const.;**
3. **the fin is cooled by a fluid with uniform temperature $t_f$ = const., convection coefficient is constant on the entire fin surface, $\alpha$ = const.;**
4. **the temperature of fin's base is uniform, there are no contact heat resistances between fin and support wall;**
5. **the thickness of the fin is small comparing to its height so that temperature gradients can be neglected;**
6. **there are no interior heat sources in the fin, $q_v$ = 0.**

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Based on these hypotheses heat transfer through fin will be unidirectional conduction with convection.

*The variable transversal section fin.*

It is considered a fin with variable transversal section

**$S = S(x)$**

and variable perimeter

**$P = P(x)$,**

in contact with a fluid with temperature

**$t_f$ = constant**

and its convection coefficient

**$\alpha$= constant**

In a certain transversal section, including its lateral perimeter, the fin's temperature is the same:

**$t = t(x) > t_f$.**

The temperature of fin's base is **$t_0$ = constant.**

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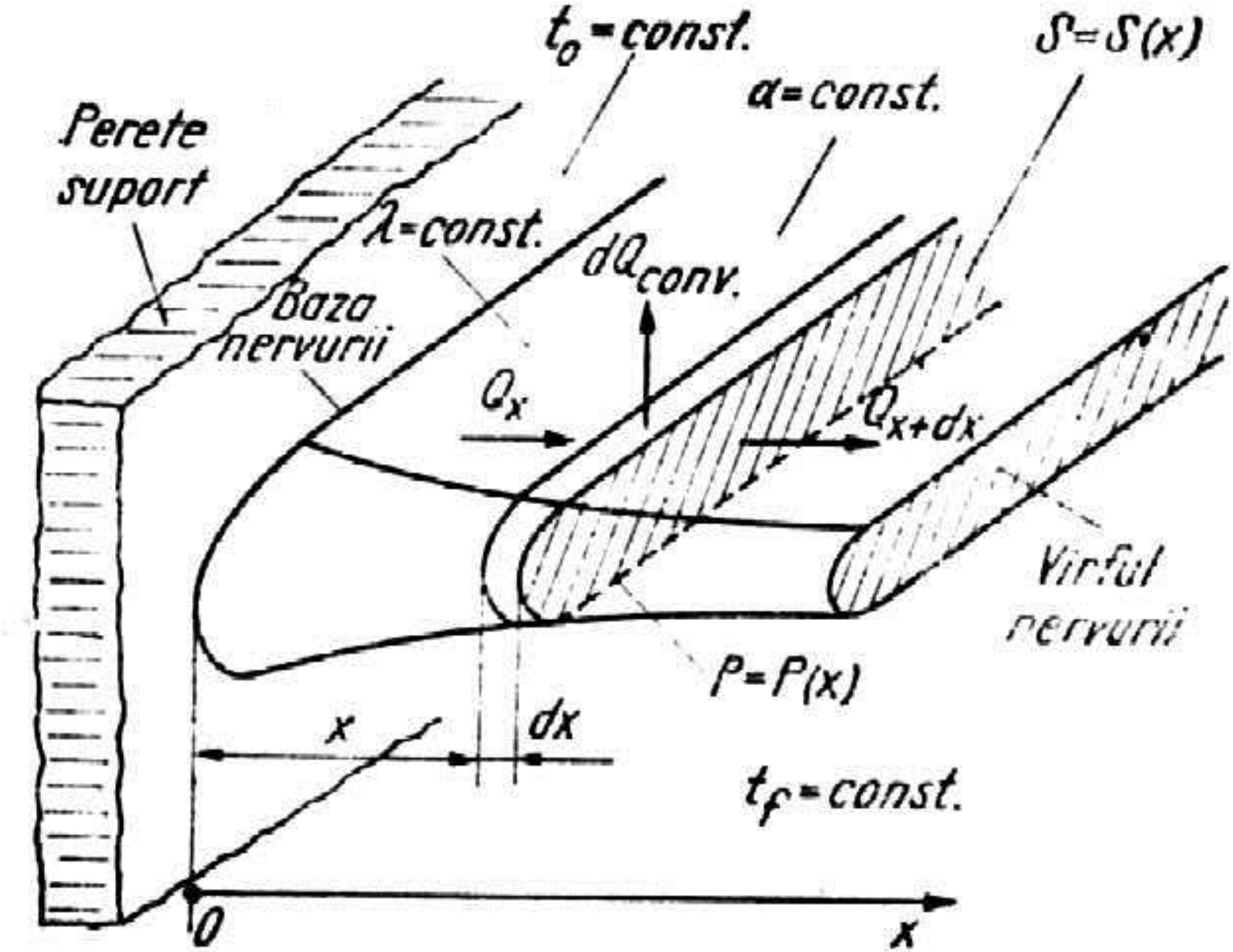


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For volume element of dx thickness from the rib it can be written the next thermal balance:

$Q_x = Q_{x+dx} + dQ_{conv}$,

where: $Q_x$ is heat flux that crosses x plan;

$Q_{x+dx}$ – heat flux that crosses x+dx plan;

$dQ_{conv}$ – heat flux transmitted to the fluid through convection.

$$Q_x = -\lambda S \frac{dt}{dx}$$

$$Q_{x+dx} = Q_x + \frac{dQ_x}{dx} dx = -\lambda S \frac{dt}{dx} - \frac{d}{dx}\left(\lambda S \frac{dt}{dx}\right) dx =$$

$$= -\lambda S \frac{dt}{dx} - \left(\lambda S \frac{d^2 t}{dx^2} + \lambda \frac{dS}{dx}\frac{dt}{dx}\right) dx$$

$$dQ_{conv} = \alpha \, P \, dx \, ( t - t_f )$$

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It is obtained the differential equation after the balance calculus:

$$\frac{d^2t}{dx^2}+\frac{1}{S}\frac{dS}{dx}\frac{dt}{dx}-\frac{\alpha P}{\lambda S}(t-t_f)=0\,.$$

If we introduce variable exchange $\theta = t - t_f$, where $\theta$ represents temperature excess between wall and fluid in $^0$C, and the report is noted as $m^2 = \alpha P / \lambda S$, ( $m^{-2}$ ); where:

$$m=m(x)=+\sqrt{\frac{\alpha P}{\lambda S}}\,,\ (\mathrm{m}^{-1}).$$

The differential equation gets the general form

$$\frac{d^2\theta}{dx^2}+\frac{1}{S}\frac{dS}{dx}\frac{d\theta}{dx}-m^2\theta=0\,.$$

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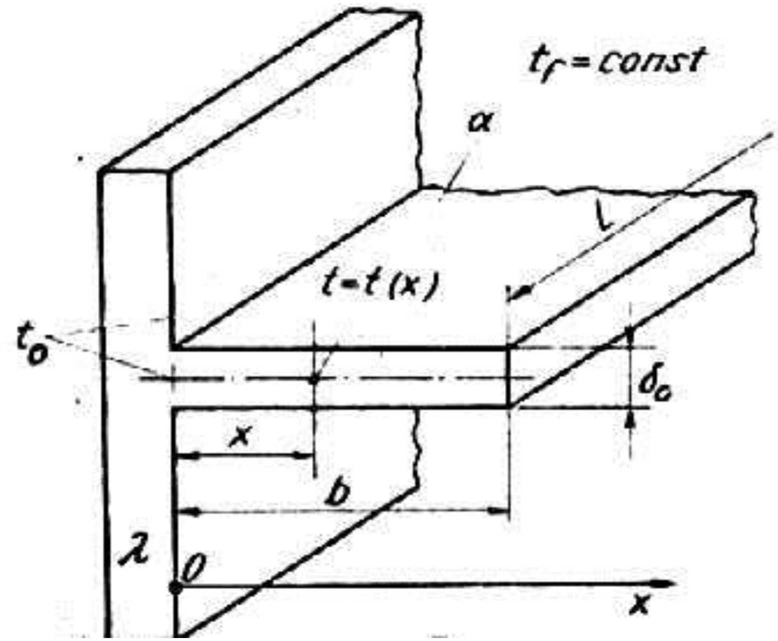
*The constant transversal section rib.* For this type belongs the straight rib with constant thickness with rectangular profile. For it, S = const., so that differential equation has the form

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0.$$




Where, with the figure notations, $m = \sqrt{\frac{2\alpha}{\lambda\delta_0}}$, iar $\theta = t - t_f$.

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## *The optimum profile*

Usually, the fins are made of materials with high thermal conductivities or corrosion resistant, both cases being very expensive. That is why it is searched the accomplishment of some fins with minimum metal consumption for a certain quantity of heat.

The problem consists in determining the longitudinal profile of the fin so that unitary thermal flow transmitted through conduction remains constant from where it results that $d\theta/dx = C_1 = const$. So, we have $\theta = C_1x + C_2$, respectively a linear variation of the difference between lateral surface and fluid temperatures.

The only longitudinal fin that has a linear distribution of temperature difference $\theta$ is concave parabolic fin which fulfills the condition of minimum material consumption.

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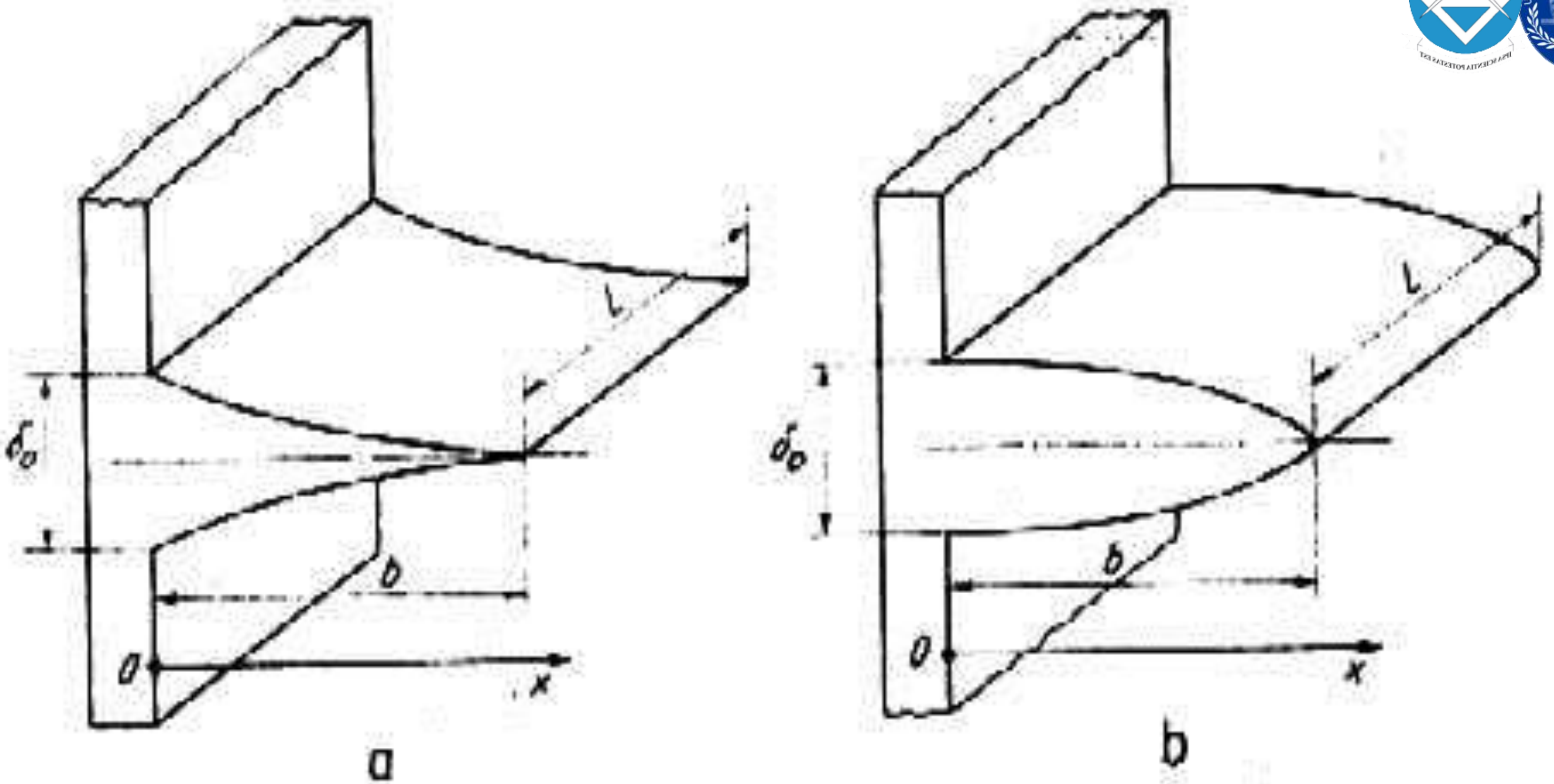


Technologically the execution of a longitudinal concave fin is difficult. In addition this profile has a low mechanical resistance. Taking into consideration that weight difference between a concave fin and a triangular one is very small, the latest being easy to realize, it can be accepted for practice use a triangular fin as an optimum form. Also from resistance motives the triangular fin is modified as a trapezoidal fin.


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*Radial fins.* In the general case of a radial fin with a certain profile, the differential equation of temperature distribution in the fin is similarly fixed by getting

$$\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}+\frac{1}{y}\frac{dy}{dr}\frac{d\theta}{dr}-\frac{\alpha}{\lambda y}\theta=0$$

where y = y (r) is fin’s thickness variable with r radius.

*Acicular fins (bar type).* Acicular fins are longitudinal fins bar type with finite dimensions of transversal section small in comparison with fin’s height.

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# APLICATIONS OF CONVECTIVE + RADIATIVE HEAT TRANSFER IN HEATED ENCLOSURES

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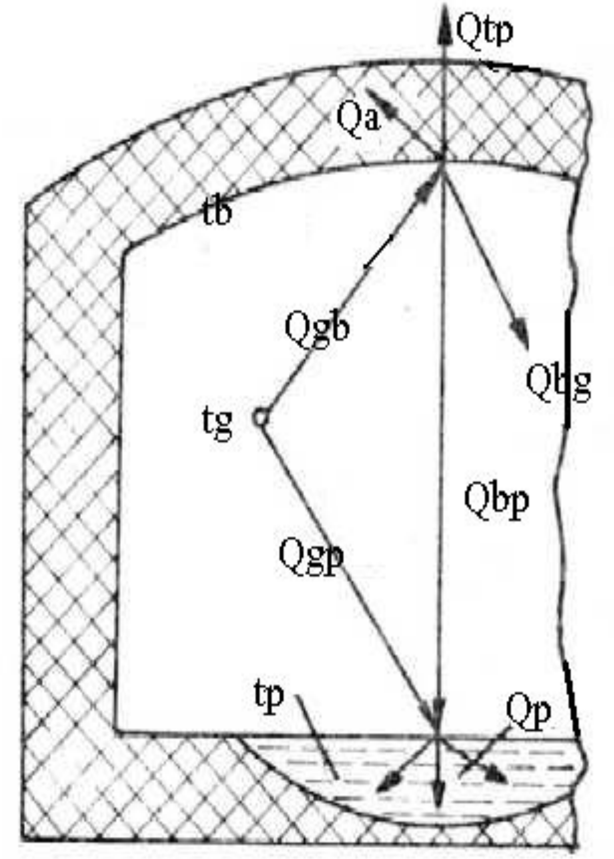
In this case, let us consider the heat transfer in a heated enclosure, with direct application to industrial furnaces.

Let us make some notations:

- ***$t_g$ – gas temperature;***
- ***$t_b$ – walls temperature***
- ***$t_p$ – part temperature;***
- ***$Q_p$ – part heat***
- ***$Q_{gp}$ – radiation+convection heat from gases to part;***
- ***$Q_{gb}$ - radiation+convection heat from gases to walls;***
- ***$Q_{bg}$ - radiation heat from walls to gases;***
- ***$Q_{bp} = Q_{gb} - Q_{bg}$ - radiation heat from wall to part;***
- ***$Q_a$, $Q_{tp}$ – heat losses through walls.***

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In an industrial heating process, the parts heating respect the relation:

$$Q_p = Q_{gp} + Q_{bp}$$

This heat received is the basis equation for designing the equipment because it gets information about the time needed for completing the process

For calculating $Q_p$ it need further notations:

**$S_b$ – walls inner surface;**
**$S_p$ – parts exterior surface;**
**$\alpha_b$ – convection coefficient towards walls;**
**$\alpha_p$ - convection coefficient towards parts;**
**$\alpha_{gb}$ - radiation coefficient between gases and walls ;**
**$\alpha_{bp}$ - radiation coefficient between walls and parts;**
**$\alpha_{gp}$ - radiation coefficient between gases and parts;**
**$\varepsilon_b$ – walls emission coefficient;**
**$\varepsilon_p$ – parts emission coefficient.**

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The heat received by the walls from gases can be:

$$Q_{gb} = S_b \cdot \alpha_b \cdot (t_g - t_b) + S_b \cdot \alpha_{gb} \cdot \varepsilon_b \cdot (t_g - t_b)$$

For a steady state regime, the heat losses are:

$Q_a = 0$

$Q_{tp} = S_b \, q_b$

where:

$q_b$ – heat losses by untights.

From figure it can say:

$Q_{gb} = Q_{tp} + Q_a + Q_{bg} + Q_{bp}$

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Furthermore:

$$Q_{gb} = S_b \cdot q_b + Q_{bg} + Q_{bp}$$

$$Q_{bg} = S_p \cdot \varepsilon_b \cdot \alpha_{gbp} \cdot (t_b - t_p)$$

$$Q'_{bp} = S_p \cdot \varepsilon_{bp} \cdot \alpha_{bp} \cdot (t_b - t_p)$$

$\alpha_{gbp}$ is the radiation coefficient of gases at walls temperature

$$\varepsilon_{bp} = \frac{1}{\dfrac{1}{\varepsilon_p} + \dfrac{S_p}{S_b}\left(\dfrac{1}{\varepsilon_b} - 1\right)}$$

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The heat radiated between parts and wall is:

$$Q_{bp} = Q'_{bp} - Q_{bg},$$

and:

$$Q_{bp} = S_p \cdot (\varepsilon_{bp} \cdot \alpha_{bp} - \varepsilon_b \cdot \alpha_{gbp}) \cdot (t_b - t_p)$$

The heat received by parts from gases is:

$$Q_{gp} = S_p \cdot \varepsilon_p \cdot \alpha_{gp} \cdot (t_g - t_p) + S_p \cdot \alpha_p \cdot (t_g - t_p)$$

Using the relation:

$$Q_p = Q_{bp} + Q_{gp}$$

it get:

$$Q_p = S_p \cdot (\varepsilon_{bp} \cdot \alpha_{bp} - \varepsilon_b \cdot \alpha_{gbp}) \cdot (t_b - t_p) + S_p \cdot (\alpha_{gp} \cdot \varepsilon_p + \alpha_p) \cdot (t_g - t_p)$$

Finally:

$$Q_p = S_p \cdot [\ (t_b - t_p) \cdot (\varepsilon_{bp} \cdot \alpha_{bp} - \varepsilon_b \cdot \alpha_{gbp}) + (t_g - t_p) \cdot (\alpha_{gp} \cdot \varepsilon_p + \alpha_p)\ ]$$

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In the case of multilayer parts, it can write:

- in rectangular parts:

$$Q_p = \frac{t_g - t_0}{\frac{1}{\alpha_i \cdot S_i} + \frac{1}{\alpha_e \cdot S_e} + \sum_{k=1}^{n} \frac{d_k}{S_k \cdot \lambda_k}}$$

- in cylindrical parts:

$$Q_p = \frac{2 \cdot \pi \cdot l (t_g - t_0)}{\frac{1}{2 \cdot \alpha_e \cdot r_e} + \frac{1}{2 \cdot \alpha_i \cdot r_i} + \sum_{k=1}^{n} \frac{1}{\lambda_k} \ln \frac{r_{k+1}}{r_k}}$$

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# THANK YOU!