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# Master Degree in Innovative Technologies in Energy Efficient Buildings for Russian & Armenian Universities and Stakeholders (MARUEEB)

Junior Intensive Course

## ENERGY EFFICIENT BUILDINGS

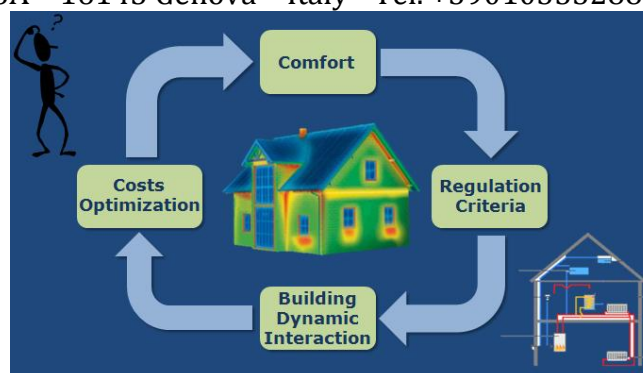
### Fundamentals of Engineering Thermodynamics

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# Fundamentals of Engineering Thermodynamics

## SUMMARY

Introduction .....	2
Basic equations for thermodynamic system design and performance analysis .....	2
Hints for problem solving and design .....	3
Control Volume balance equations.....	5
Control Volume Mass balance – Principle of mass conservation .....	5
Control volume Energy balance – The first law of thermodynamics .....	6
EFFICIENCY .....	6
Control Volume entropy balance - The second law of thermodynamics.....	7
Thermal and physical properties of materials, fluids and systems .....	8
The transport rate - closure laws .....	12
References.....	13

## Introduction

The use of the fundamental equations of the applied thermodynamics, the energetic ones and the transfer of heat and mass has a fundamental role in the definition and the quantitative evaluation of any industrial process. The deep understanding of the way a system work, based on quantitative equations, even in an approximate and one-dimensional form, that is on the base of parameters “lumped” over the control volume for the description of the transient behavior (time-varying), is an essential requirement for the system design and engineering.

The system analysis starts from the detailed lists of a system duty (user requirements) for the development of the theoretical processes and to the definition at last of the detailed lists of the components of the industrial system, with their specifications in terms working conditions, size and features. These disciplines are completed with the studies on mass and momentum transfer, which are equally needed to complete a system analysis.

In this short review the fundamental equations commonly used in the study of the industrial systems will be presented, omitting some theoretical aspects being the basic cultural knowledge of that immense Energetic and Thermodynamic discipline more commonly known as “Applied Engineering thermodynamics, thermal and fluid dynamics, heat and mass transfer”.

## Basic equations for thermodynamic system design and performance analysis

In order to correctly define a system in its main functions from the thermodynamic point of view and to determine the preliminary sizes of all its components, we must first of all correctly characterize the system, the nature of the interactions with the outside, the role of its single components and their mutual interactions inside the plant (heat exchangers, turbines, pumps, piping, valves, auxiliary components, specifications of electrical systems, measurement and control elements, and so on).

The majority of the analyses uses, more or less explicitly, the three fundamental **balance equations** (mass, energy, entropy), which must be correctly coupled the thermodynamic properties of the operating fluid in the system (gas, liquids, two-phase fluid and/or phase transition, incompressible fluids, viscous fluids, and so on), generally known with the name of **constitutive equations** or state equations. These two mains classes of equations are at last completed with the equations that describe the kinetic one of interactions of the components (or the fluid) with their respective outside (“environment”), by means of proper kinetic equations of transfer rate(of mass for osmosis, momentum for friction, energetic for heat exchange and



mechanical/electrical work, etc.). These equations, generally with the name of transport or closure equations (**transport rate equations**) are defined typically by means of experimental or empirical laws, such as the law of Fourier for the heat transfer rate, the Fick's law for the mass transfer rate, the law of Darcy for the friction effects opposing the flow of fluids in ducts and the viscosity characteristics (Newtonian, non-Newtonian) of the fluid, and so on.



## Hints for problem solving and design

In order to obtain a satisfactory and reliable understanding of the processes and a correct sizing of the systems (components) used in order to built it, it is necessary, in general terms, to perform some simple steps in succession:

1 - **Functional data:** To read carefully or to write, if not already available, a global description of the system, its functionalities and the hypotheses of operation/working choices on which the description is based.

2 - **Sketch of system:** To develop or to acquire a system diagram, also simplified, that summarizes the relative position of every component, its links to other components and to draw on the diagram the fundamental information related to heat loads, flows, operating ranges, working pressures and temperatures and so on.

3 - **Working/simplifying hypothesis:** to deliver an adequate list of the simplifying hypotheses adopted in the system study, to be able on one side to arrive quickly to an analysis of system performances, on the other side to have the possibility, if necessary, to return on the hypotheses for better define those ones needed to deepen much more the study.

4 - **Analysis criteria and tools:** to characterize the instruments (tools) necessary for the calculation and to develop the analysis of the system on the base of the hypotheses done. As far as possible avoid numerical calculations with row numbers, keeping the analytic form (or the calculation algorithm on computer in appropriate programming language) which could allows us to carry on studies of sensibility and optimization of the process itself.

5 - **Final considerations:** To drop a short relation on the acquired results, making comments on the hypotheses used, the more meaningful results and working/environmental parameters that influence the process.

6 - **Functional detailed lists:** once the analysis has been completed, it will be possible to define all the dimensional and electrical working conditions of each component (weights, blocks, systems of anchorage, combustion and other reaction emissions, raw materials, etc.). These data shall be supplied to the others members of the working group to plan all the industrial plant aspects, such as measure and automation, electrical engineering, constructions and civil works.

7 - **Iterate** on the issues above, also with the colleagues of the other teams, until the best configuration of the system is found, or the working conditions offering satisfactory results in



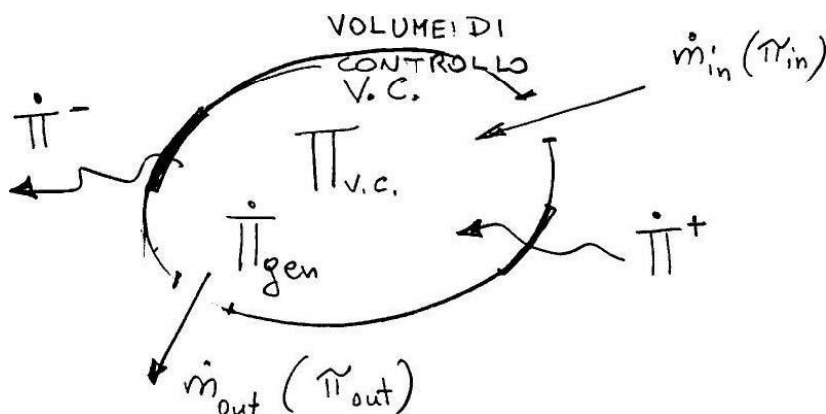
terms of efficiency, reliability, costs, are reached. Usually a trial-and-error procedure to find the best trade-off for all the requirements shall be needed: the clever development of this job favors the system deployment in sub-problems in a top-down approach, its structuring for independent, collaborating working teams and make the communications of the achieved results easier both inside and outside of the Engineering Society responsible of the project.

## THE CONTROL VOLUME APPROACH

Defined “system” the object (plant) that interests the study in question (enclosed in a *Control Volume* by means of a *Control Surface*, which can be physical or imaginary) and calling its “outside” that part of the universe that interacts with it in some meaningful way, a physical balance can be carried out of any extensive characteristic of the system (Property) by means of a quantitative evaluation of the “exchanges” between the system and its outside.

The exchanges (property transfers) take place through surfaces (interfaces) of the system especially planned to the aim on the control surface. The control surface design can favor mass or heat transfer, mechanical energy or deformation, and can be designed to avoid the flow of specific physical entities or of energy flow (e.g. thermal insulation).

Every specific component of the system (turbines, fans, heat exchangers, pumps, compressors, etc.) is planned in order to carry out at best a well defined function of interaction, favoring the evolution of the system along a given process (transformation, thermodynamic cycle, process of energy conversion, etc.).



Generally the processes having greater application interest are those repeated with continuity in the time in conditions of stationary regime (cyclic processes). In this case the variable “time” does not have a fundamental role, if not in the transition from the extensive amounts exchanged to those exchanged in the unit of time (energy-Power, heat transfer thermal heat-transfer-rate, mass transfer – mass flow rate, and so on). However in several applications (for example in the analysis of the control systems, in the intermittent processes, in the analysis of thermal start up of a car or a system) the “lumped” analysis of the systems in transient conditions can be of great interest. The term “lumped” means that property values in the control volume are assumed to be uniform, considering a single numerical value representative of the whole control volume. Such a value is calculated by means of a weighted average of the property values measured in various spatial points of the system under investigation.

In the case such averages could not be assumed as meaningful (may be because of strong gradients in the system being a fundamental operating requirement of the process, or because the system is built by means of more independent elements with different property values) a good thermodynamic analysis will be any way possible, decomposing the system in more subsystems of smaller complexity; reference will be made then to the *Operating Unit* or, going even more into the details, to the single component of the system. Generally the systems are described by a number of thermodynamic features, called property, that satisfy the characteristics of state-functions and can be intensive that is not proportional to system mass ( $T, p, w$ ), or extensive, proportional to system mass ( $M, V, U$ ,



H, S, Ec, Ep). If the system shows uniform properties, to an extensive quantity can be associated its homologous intensive value, dividing for the mass value M to which the extensive value is referred ( $v=V/M$ ,  $u=U/M$ ,  $e_c=w_2/2$ ,  $e_p=gz$ , etc.).

## Control Volume balance equations

For any *property* of a system and in any operating condition (also when there is a matter exchange, that is mass flow), let us call  $\Pi$  any extensive property and  $\pi = \Pi/M$  the correspondent intensive property (with proper meaning and operating characteristics). We define moreover  $\dot{m} = \delta M / \Delta \tau$  the flow of mass in the unit of time (*flow rate m*) through a given portion (opening) of the control surface of the system, as briefly sketched in the figure. Every entering or outgoing flow rate under investigation will have its specific intensive value of the property in question,  $\pi$ .

In the figure  $\Pi_{v.c.}$  is the extensive property that we consider uniform in the control volume,  $\pi_{in}$  and  $\pi_{us}$  are respectively the contributions of variation of the specific property at the inlet and outlet of the flow rate,  $\dot{\Pi}_-$  e  $\dot{\Pi}_+$  are the diffusive contributions (osmotic flows) across the control surface and finally  $\dot{\Pi}_{gen}$  is the internal generation rate of the property under investigation.

We

We can always write the control-volume property balance based on the property flow rates as follows:

$$\frac{d\Pi_{v.c.}}{d\tau} = \dot{\Pi}_+ - \dot{\Pi}_- + \sum_{in} (\dot{m}_{in} \cdot \pi_{in}) - \sum_{us} (\dot{m}_{us} \cdot \pi_{us}) + \dot{\Pi}_{gen}$$

## Control Volume Mass balance – Principle of mass conservation

Let us consider for example the property mass M: in all “the common” situations (we do not deal with nuclear or chemical reactions here) the mass is not created neither it is destroyed and this claim corresponds to, or *principle of conservation of the mass*, in the sense of a characteristic of the physical property “mass M of the system”. If we place  $\Pi_{v.c.} = M_{v.c.}$  as the mass of the control volume and we neglect the processes of mass diffusion through the control surface (osmotic flows), such as the phenomena of molecular diffusion (and therefore assuming  $\dot{\Pi}_+ = 0$  and  $\dot{\Pi}_- = 0$ ) we will have:

$$\frac{dM_{v.c.}}{dt} = \sum_{in} \dot{m}_{in} \cdot 1 - \sum_{us} \dot{m}_{us} \cdot 1 + 0$$

THAT IS:

$$\frac{dM_{v.c.}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{us} \dot{m}_{us}$$

Note: if nuclear reactions *are present*, the equivalence mass-energy must be involved. If chemical reactions *are present*, we must introduce a specific *mass balance* for each chemical species involved. Finally if also osmotic flow rates are involved (Biology, but also processes of desalination for inverse osmosis RO-Reverse Osmosis) the terms of flow of matter through membranes *shall be* carefully estimated, on the base of *proper mass transfer* equations, and inserted to the place of the first two terms of the *mass-balance* equation.



## Control volume Energy balance – The first law of thermodynamics

Let us consider the extensive property  $\Pi_{vc}$  of a thermodynamic system to represent the energy level of its mass  $M$  inside the control volume, hence:

$$\Pi = E_{vc} = U + \frac{1}{2} \cdot M \cdot w^2 + M \cdot g \cdot z + \dots \text{ or in intensive form } \pi = e_{vc} = u + \frac{1}{2} \cdot w^2 + g \cdot z + \dots$$

The general balance expressing the variation of the inner energy in the unit of time, in the terms of the sum of all the thermal fluxes that the system exchanges with outside ( $\dot{Q}$ ), of the sum of all the exchanges of “work” (mechanical, electrical, ...) in the unit of time ( $\dot{L}$ , that is a mechanical power) and of the mass flows through the control surface (inlet and outlet) is:

$$\frac{dE_{vc}}{d\tau} = \dot{Q} - \dot{L} + \sum_{in} \dot{m}_{in} \cdot e_{in} - \sum_{us} \dot{m}_{us} \cdot e_{us} + \dot{E}_{gen}$$

If the energy principle is assumed, (with no nuclear or chemical reactions) we have:

$$\Leftrightarrow \frac{dE_{vc}}{d\tau} = \dot{Q} - \dot{L} + \sum_{in} \dot{m}_{in} \cdot e_{in} - \sum_{us} \dot{m}_{us} \cdot e_{us}$$

Splitting the work into its flow component, (the power needed to assure the mass flow rate across the control volume) and the shaft + volume change work (the so called useful external work, usually obtained by means of a shaft inside the system or sometimes by deformation of the control volume, such as in heart beat) we can introduce the thermodynamic property enthalpy  $h = u + pv$  [J/kg] to achieve:

$$\frac{dE_{vc}}{d\tau} = \dot{Q} - \dot{L}_e + \sum_{in} \dot{m}_{in} \cdot \left( p \cdot v + u + \frac{1}{2} \cdot w^2 + g \cdot z \right) - \sum_{us} \dot{m}_{us} \cdot \left( p \cdot v + u + \frac{1}{2} \cdot w^2 + g \cdot z \right)$$

$$\boxed{\frac{dE_{vc}}{d\tau} = \dot{Q} - \dot{L}_e + \sum_{in} \dot{m}_{in} \cdot \left( h + \frac{1}{2} \cdot w^2 + g \cdot z \right) - \sum_{us} \dot{m}_{us} \cdot \left( h + \frac{1}{2} \cdot w^2 + g \cdot z \right)}$$

$$\boxed{\frac{dE_{vc}}{d\tau} = \dot{Q} - \dot{L}_e + \sum_{in} \dot{m}_{in} \cdot (h_t) - \sum_{us} \dot{m}_{us} \cdot (h_t)}$$

$$h_t = \left( h + \frac{1}{2} \cdot w^2 + g \cdot z \right)$$

In these equations the fluid property  $h_t$  is also called “total enthalpy” of the fluid, that is a state-property often used in the study of turbo machinery, be it for turbines, compressors, fans or pumps.

State properties must be calculated on the basis of actual pressure, temperature, quality (if relevant) values.

## EFFICIENCY

Any system able to perform a useful effect (usually required and defined by the user) will work asking a consumption of resources, corresponding to some “operation cost”. Based on the first principle of the thermodynamics is possible to calculate the efficiency of first principle  $\eta_1$ , defined in the following way:





$$\eta_I = \frac{\text{actual useful effect}}{\text{system "COST" needed}} \quad (\eta_I < 1 \text{ if numerator and denominator are consistent}).$$

The two indicated quantities generally are homogenous from the physical and dimensional point of view, so that  $\eta_I$  is a non-dimensional number usually less than 1. If the “physical nature” or measuring units of numerator and denominator are different (as it could be for heat/job) the first-principle efficiency  $\eta_I$  can assume indeed any numerical value.

From the operating point of view another way exists to reason, thinking on the possibilities or capabilities that a system has to work “well or not”. For each system, indeed, it could be useful to know if the actual operating conditions correspond to “best possible conditions”. For instance the human body has by no means high first-law efficiency  $\eta_I$ , but its complex processes assure the best possible performance, given the working temperatures of the body and the needs to perform mechanical work.

Let us call  $(\eta_I)_{\max}$  the maximum first-law efficiency theoretically achievable by means of one potentially ideal machine operating in the best ideal working conditions (that is the efficiency of the operating ideal system in ideal conditions). The conditions of *ideal system* and *ideal conditions* are both necessary in order to obtain the maximum, from the point of view of the second-law efficiency (that is the efficiency based on the second law of thermodynamic analysis) in the form:

$$\eta_{II} = \eta_I / (\eta_I)_{\max}$$

This definition lead historically to the concept of “availability”, and it is easy to understand how it will always be less than unity, since no ideal machine or ideal condition can be possible in actual world

## Control Volume entropy balance - The second law of thermodynamics

The above considerations are summarized in the second law of thermodynamics principle, leading to the definition of a particular property, the *entropy*  $S$  [ J / K ] (whose corresponding intensive property is  $s$  [ J / (kgK) ]). If we consider only thermal and mechanical systems (no magnetic, electrical or nuclear) the entropy variation of a control volume can be easily correlated, in an operative statement, to heat transfer across the control surface and to its working temperature:

$$dS = \left( \frac{\delta Q}{T} \right)_{SC} + \delta S_{\text{gen}}$$

The term  $\delta S_{\text{gen}}$ , which is not a “property”, is called *entropy production* of the process. This is a characteristic qualifying the coupled behavior of the system and its interactions with the environment: the “worse” is the process, the higher is  $\delta S_{\text{gen}}$  will be.

In terms of general control-volume balance equations we have (II = S):

$$\frac{dS_{VC}}{d\tau} = \sum_{SC} \left( \frac{\dot{Q}}{T} \right) + \sum_{in} \dot{m}_{in} \cdot s_{in} - \sum_{us} \dot{m}_{us} \cdot s_{us} + \delta \dot{S}_{\text{gen}}$$

In this relation we have indicated with  $\delta \dot{S}_{\text{gen}}$  the rate of entropy internal generation, while  $s_{in}$  and  $s_{us}$  are the specific entropy of the fluid flowing at the inlet and outlet of the control volume respectively. It is worth noting that entropy is not a “conservative” property. While the equations of conservation of mass and energy, can be used as algebraic or first-order differential equations in closed form, the second principle of thermodynamics

cannot be turned in an equality to zero, due to the constraint

$$\delta \dot{S}_{\text{gen}} \geq 0$$

$$\delta \dot{S}_{\text{gen}} \geq 0$$



However it constitutes a possible equation only in the case of ideal behavior  $\delta \dot{S}_{gen} \equiv 0$ .

The entropy balance equation in actual studies is therefore an inequality; for this reason it has not so great practical application. However it can have great technological interest, since using the assumption  $\delta \dot{S}_{gen} \equiv 0$  we can find the conditions of maximum efficiency of a system, the unsurpassable limit beyond which the technology, regardless how much sophisticated and expensive it is, cannot go.

In conclusion the first principle-efficiency calculations ( $\eta_I$ ) are useful in energy and operating costs analyses, while the efficiency calculations based on the second principle of thermodynamics [ $\eta_{II} = \eta_I / (\eta_I)_{max}$  to be maximized, that is entropy generation minimization) are more useful in order to estimate the technological level of the equipment, the components, the systems and for generic evaluation of environmental impact (not indeed all-life-cycle analysis, but potential plant environmental impact during operation).

## Thermal and physical properties of materials, fluids and systems

The balance equations described so far can be applied to any possible thermodynamic system, offering therefore the solution of an immense class of problems in the industrial plants technology. They all concur to the definition of the nominal working conditions of the several subsystems and guide the technical choice of the components of each system, needed for given duties and performance.

Integration (numerical, algebraic, analytics or computer-aided) of such equations requires the preliminary knowledge of the properties of the operating fluids and materials that are involved, in order to be able to correlate the various properties used in conservation equations ( $p, T, u, h, s, v, \dots$ ) to the measurable fundamental properties ( $p, T, v, \text{mass}$ ).

The discipline involved in these aspects is based on the early studies of Maxwell, Gibbs, Helmholtz, has a long-time history and is nowadays quite well developed and established under the name of *thermo physical properties of materials*. A lot of algorithms of calculation are available in well-structured programs, very consolidated and reliable (see literature references: NIST, EES,...).



In practice relations are needed to describe the behavior of the fluid in the several thermodynamic conditions, allowing the calculation of their intensive properties to be performed, based at least on the values of two assigned, independent numerical values to each property in each thermodynamic state. Such studies leads to

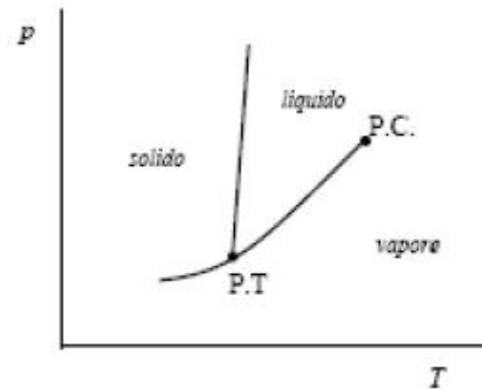


Fig. 6.3. Diagramma delle fasi per sostanze che solidificando si contraggono.

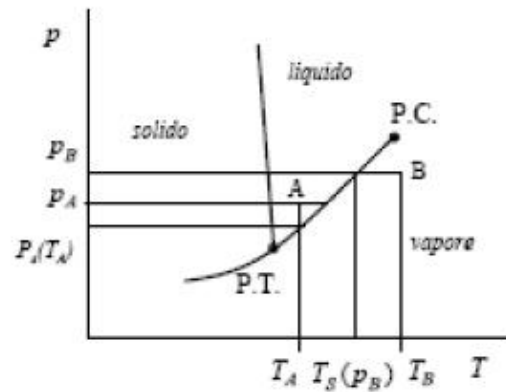


Fig. 6.4. Diagramma delle fasi per sostanze che, come l'acqua, solidificando si dilatano.

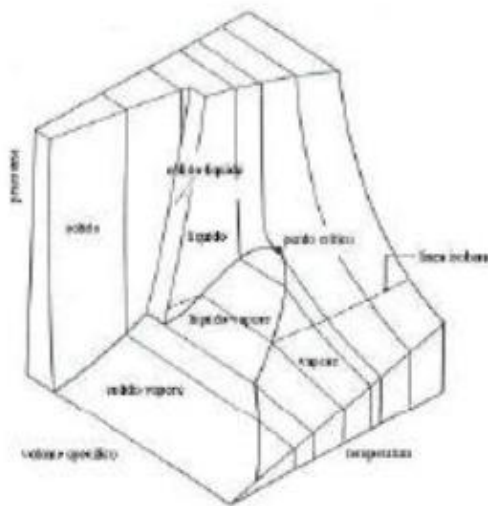


Fig. 6.1. Superficie  $p, v, T$  tipica della maggior parte delle sostanze che, solidificando, si contraggono.

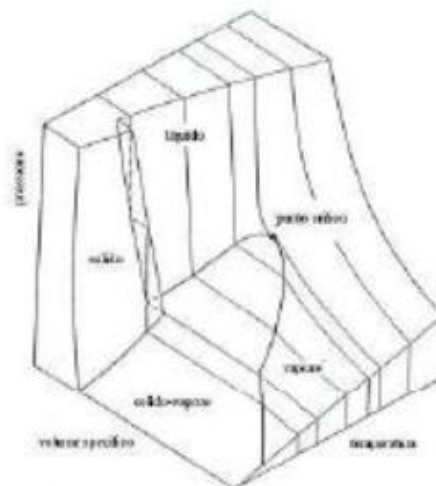


Fig. 6.2. Superficie  $p, v, T$  caratteristica di una sostanza che, come l'acqua, solidificando si dilata.

the quantitative definition of the diagrams of the phases and to assign numerical values to each property in each thermodynamic state of operation.

As an example the figure reports the case of “simple fluids” consisting of a pure substance. In this simple case it is possible to write, in explicit or implicit form, a constitutive equation of the type  $f(p, T, v) = 0$ .

These experimental diagrams are the basis to calculate, following given controlled thermodynamic process (imposed heat transfer rate, adiabatic, with assigned and known exchanges of work or mechanical power, at constant pressure or specific volume, and so on) to all the other thermodynamic data (entropy, enthalpy, .....

As an example the figures shows the T-S plane of water.

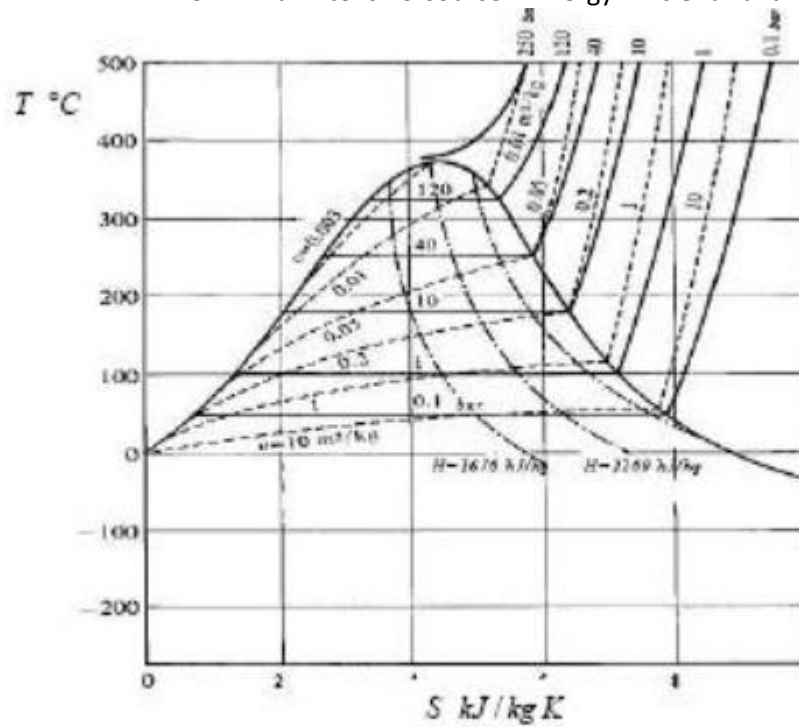


Fig. 6.9. Diagramma entropico dell'acqua, con l'origine nello stato di riferimento: punto triplo, liquido saturo.

Also the p-h plane (in refrigeration technology) and h-s planes (in energy conversion studies) are widely used.

Normally the use of the thermodynamic diagrams is possible for the simpler cases. In the cases more complex complete programs like NIST or EES can be used, since they quickly to carry out the calculation of thermodynamic processes in many operating conditions. When the thermodynamic process is much complex (many components, many subsystems that interact in between ) the study can be carried out making use of complete commercial programs of calculation, more difficult to be used, such as for example PROVISION or THERMOFLOW.

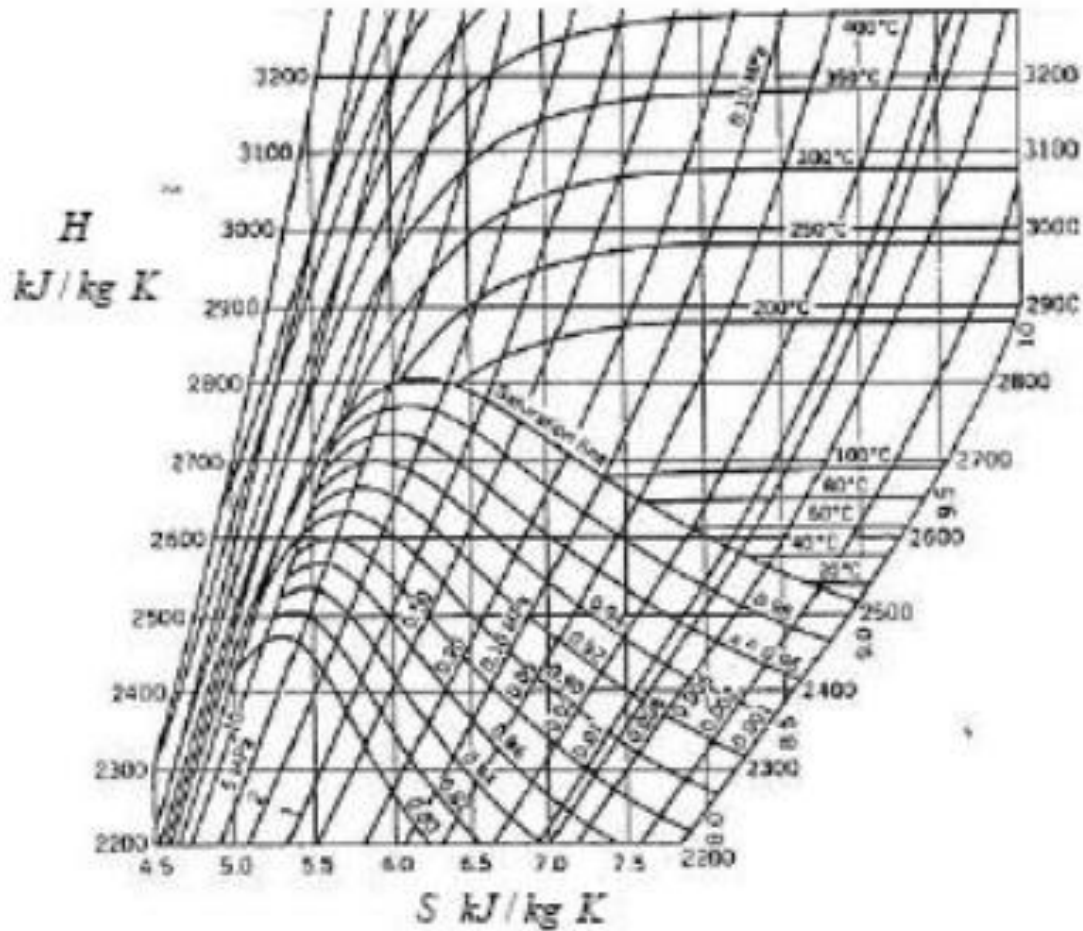
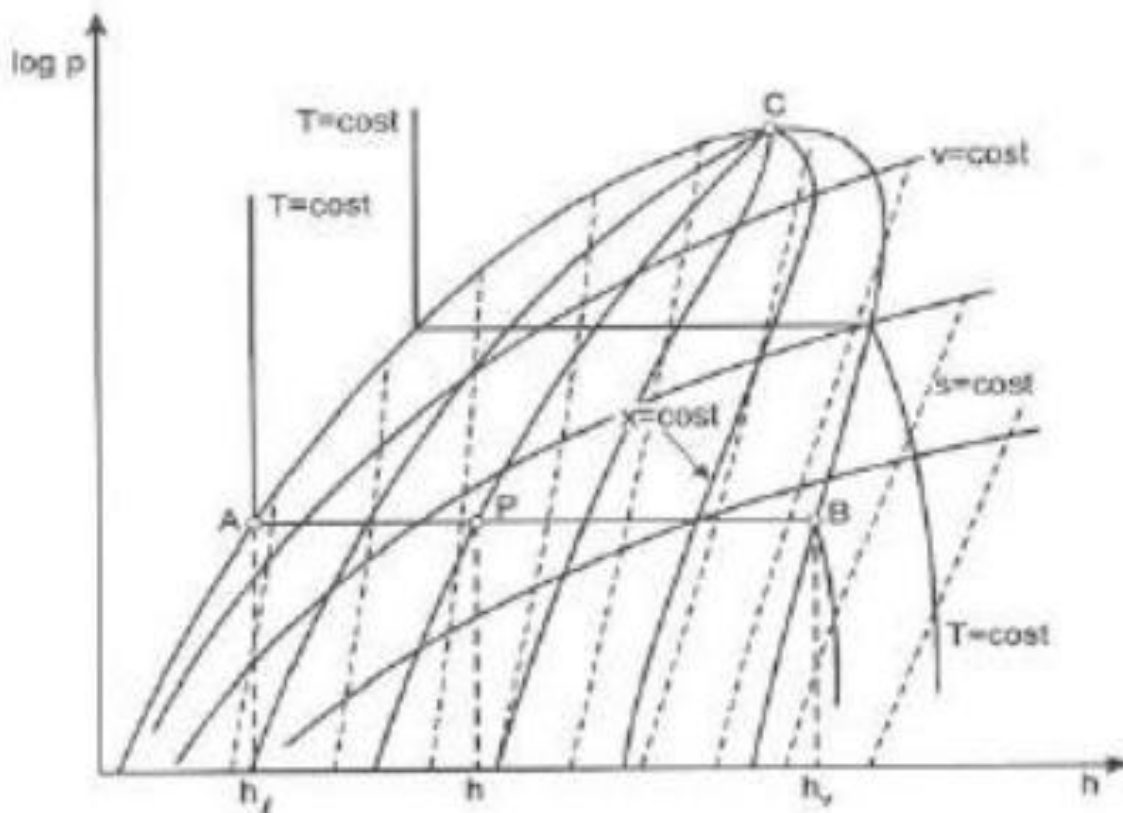


Fig. 6.10. Diagramma entalpico o di Mollier ( $H, S$ ) dell'acqua.



11. Diagramma ( $p, H$ ) di un generico refrigerante, con scala logaritmica della pressione.



## The transport rate - closure laws

The brief overview about the conservation laws (or rather the *balance equations*) of the systems and the structure of the *constitutive equations* of a thermodynamic fluid, must be completed with the laws that control the interactions of the system components with each-other and the environment. These equations are defined by the characteristics of “transport” across the control surfaces and are called “transport rate equations” or “closure laws”.

This last passage, enough simple in its general formulations, often becomes the greater source of uncertainty in the correct design of the systems, since it is mainly based on empirical/experimental knowledge and is also the greater source of technological research & development in the process industry. While in fact the equations seen so far are based on physical laws, able to impose very defined links between the thermodynamics characteristics of each system, this last aspect have mostly empiricist-experimental characteristics, in order to describe “kinetic” of the processes of exchange (that is interaction) between the systems and their outside.

Dynamic equations of transfer rate (“closure laws”)	<b>Empirical laws</b>
	Fick’s law (diffusion)
	Fourier’s law (conduction heat transfer)
	Newton’s law (convection heat transfer)
	Boltzman’s and Plank’s laws (radiation heat transfer)
	Darcy’s law (fluid friction, Moody diagram)

This aspect often constitutes the true bottle-neck through which every design procedure must go trough, in order to guarantee that the planned systems have the just sizes (neither too much great, neither too much small ones) in order to carry out the functions (heat transfer, mechanical power, pumping power, etc.) demanded by the users during operation.

While the thermodynamics establishes a link in some way static between the several system properties, and in particular between the variations of  $T$ ,  $p$ ,  $v$  of a system and the entity of the energy transfer rates needed to perform the desired processes, the transport equations put explicit reference to the fundamental element of every productive cycle: **time**. Any amount of heat transfer for example implies the existence of a dedicated surface of the interface (physically the component called heat exchanger) that assures that such an energy transfer will be completed in a reasonable time. This is the link among the processes (*thermodynamic*) and the powers (*transfer rates*) and the size (*geometric dimensions*) of each single plant component.

The more effective will be the transfer rates (thermal, of mass, electrical current,...) the smaller will be, for the same power needs and performance duties, the component..



Table 1.1 Transport rate equations\*

Transport process	Mass	Momentum	Heat	Electricity
Property transferred $\phi$	Mass $N$ , (kg/s)	Shear stress $\tau$ (N/m <sup>2</sup> )	Heat $q$ , (W)	Current $i$ (amp)
Potential $\Phi$	Mass concentration (or density) $C$ , (kg/m <sup>3</sup> )	Momentum concentration $\rho$ (kg/m <sup>3</sup> ) or velocity $v$ (m/s)	Energy concentration $e$ (kJ/m <sup>3</sup> ) or temperature $T$ (°C)	$EC^*$ (volt farad/m) or voltage $E$ (volt)
Diffusion type Diffusivity or conductivity	Mass diffusivity $D_{12}$ (m <sup>2</sup> /s)	Kinematic viscosity $\nu$ (m <sup>2</sup> /s) or absolute viscosity $\mu$ (kg/s m)	Thermal diffusivity $\alpha$ (m <sup>2</sup> /s) or thermal conductivity $k$ (W/m °C)	$\frac{1}{R^*C^*}$ (m/Ω farad) or $\frac{1}{R^*}$ (m/Ω)
Rate equation as a function of potential gradient	Fick's first law $\frac{N_1}{A} = -D_{12} \frac{dC_1}{dx}$ (1.21)	Newton's law of viscosity $\tau = -\mu \frac{dv}{dy}$ (1.22) or $\tau = -\mu \frac{dv}{dy}$ (1.23)	Fourier's conduction equation $\frac{q_1}{A} = -k \frac{dT}{dx}$ (1.24) or $\frac{q_1}{A} = -k \frac{dT}{dx}$ (1.25)	$i = -\frac{1}{R^*C^*} \frac{d(EC^*)}{dy}$ or $i = -\frac{1}{R^*} \frac{dE}{dy}$ (1.27)
Convective type Coefficient	Mass transfer coefficient $h_m$ (m/s)	Friction coefficient $f$ (dimensionless)	Heat transfer coefficient $h$ (W/m <sup>2</sup> °C) $A_c = \frac{k}{L}$ for conduction $A_c = h_c$ for convection $A_c = \frac{F\sigma(T_1 - T_2)}{T_1 - T_2}$ for radiation	$\frac{1}{R(\Omega)}$
Rate equation as a function of potential difference	$\frac{N_1}{A} = h_m C_1$ (1.28)	$\tau = f \frac{\rho v^2}{2}$ for turbulent flow (1.26)	$\frac{q}{A} = h\Delta T$ (1.29) a. Fourier's conduction equation b. Newton's law of cooling c. Stefan-Boltzmann equation $q = \sigma T^4$	Ohm's equation for electrical resistance $i = \frac{3E}{R}$ (1.28) For a capacitor (C): $i = C \frac{dE}{dt}$ (1.29) For an inductor (L): $i = \frac{1}{L} \int E dt$ (1.30)

\*E, voltage; C, capacitance; R, resistance; C\*, conductivity/length; R\*, resistance/length.

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